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Aircraft Nonminimum Phase Control in Dynamic Sliding Manifolds

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The approximate causal nonminimum phase output tracking problem is considered for an F-16 nonlinear six-degree-of-freedom model and addressed via sliding mode control. Asymptotic output tracking-error dynamics with desired eigenvalue placement are provided in case of tracking causal reference output profile with a finite number of nonzero time derivatives (piecewise polynomial spline model) in the presence of unmatched disturbances of the same kind. A complete constructive algorithm for tracking controller design is built for a class of uncertain nonlinear multi-input/multi-output systems with known linear unstable internal dynamics. An analysis is made of the issue that the given nonlinear aircraft model yields to the approach developed.

Nomenclature

p	=	roll rate, rad/s
q	=	pitch rate, rad/s
r	=	yaw rate, rad/s
u_1, u_2, u_3	=	control commands to elevator, ailerons, and rudder actuators
α	=	angle of attack, rad
β	=	sideslip angle, rad
γ	=	flight-path angle, rad
$\delta_e, \delta_a, \delta_r$	=	elevator, ailerons, rudder deflections, rad
θ	=	pitch angle, rad
φ	=	roll angle, rad

I. Introduction

NONMINIMUM phase output tracking is an interesting, real life control objective that has been extensively studied recently. Sustained attention has been given to it as being applied to nonlinear aircraft control systems. It is known that a nonlinear control system is of nonminimum phase if its internal or zero dynamics[1] are unstable. The nonminimum phase nature of a plant restricts application of the powerful nonlinear control techniques such as feedback linearization control[1] and sliding mode control (SMC).[2–4] Quite a few works address this problem for various nonlinear aircraft models with different approximation techniques.[5–11] The main feature of the techniques proposed in the Refs. 5 and 8 is that they allow for slightly nonminimum phase systems. Another approach in Ref. 11 investigates the problem for the differentially flat systems.[12] A number of regulation schemes with output redefinition have been proposed in Refs. 6 and 13. This output redefinition leads to a system with stable zero dynamics (minimum phase). In Ref. 13, an SMC algorithm with output redefinition is studied. Two other techniques, which address nonminimum phase tracking via dynamic SMC, are presented in Refs. 9 and 10 where the term dynamic SMC is used to define a dynamic controller described by ordinary differential equations with a discontinuous right-hand side. The vector of conventional sliding surface quantities [for a single-input/single-output (SISO) case, it is a scalar quantity] depends on the system state variables algebraically, and the surface can be defined as a static one for that case. To eliminate chattering, the work in Ref. 9 employs a linear high-gain alternative to the relay element (saturation function),[14] whereas the work in Ref. 10 uses an entirely continuous control signal, which comes from the output of a dynamic controller, such that there is no direct discontinuous component in the control signal.

In this paper we approach the approximate causal (given in real time) output tracking; the reference profile with the finite number of nonzero time derivatives (piecewise presented by a polynomial spline) in the presence of the unmatched disturbance of the same property will be asymptotically followed. This work employs SMC,[2–4] in particular the dynamic sliding manifold (DSM)[15,16] technique. In the DSM technique the vector of sliding surface quantities dynamically depends on the system state variables, and, unlike that of other works,[9,10] the control law is a static one. To avoid chattering, this control law is presented by a nonlinear continuous function of sliding surface quantities that provides for finite time convergence to a small domain of attraction around the sliding surface; the size of this domain can be specified to be arbitrarily small.[17]

SMC design starts with constructing a sliding surface (or sliding manifold) $\sigma = \sigma(x, t)$ in the system state space $x \in \Re^n$, and the system motion along the surface $\sigma = 0$ (called the sliding mode) is considered as desirable in some sense. It is proved to be robust to the system parameter uncertainty and matched unknown bounded disturbances. Usually, to provide the existence of conventional sliding mode it is enough for the first time derivative of the sliding surface quantity σ to be proportional to the control. The system motion on this surface is the goal of control synthesis. In a nonminimum phase case the conventional SMC appears to be nonrealizable because its equivalent control term tends to infinity.[2,16]

This work is an extension of the work in Ref. 16 to the MIMO case and the state-space representation, applied to a conventional takeoff and landing aircraft model.[18] As to the approach developed, there is no output redefinition or solution of unstable differential equation required. Addressing nonlinear nonminimum phase output tracking, the sliding mode controller based on a DSM joins features of a conventional sliding mode controller (insensitivity to matched disturbances and nonlinearities) and a conventional dynamic compensator (accommodation to unmatched disturbances).

The structure of the paper is as follows. An aircraft mathematical model is presented in Sec. II, and the control goal is formulated. Section III discusses the issue of nonminimum phase output for the presented MIMO system. To address nonminimum phase nature of this plant, it is necessary to stabilize closed-loop zero (internal) dynamics in the longitudinal channel. Section IV addresses the problem of nonminimum phase output tracking for a class of nonlinear MIMO systems via DSM design. Nonminimum phase output tracking is not possible via conventional SMC design. In Sec. V, the general technique developed in Sec. IV is applied to the flight-path angle tracking in the system described in Sec. II. A DSM-based tracking controller for the lateral channel is designed in Sec. VI; the discussion on simulation results is presented as well. Conclusions are made in Sec. VII.

II. Problem Formulation

The nonlinear mathematical flight model of an F-16 jet fighter at Mach = 0.7, $h = 10{,}000$ ft, $\alpha_{\text{trim}} = \theta_{\text{trim}} = 0.1068$ rad, $\delta_{e_{\text{trim}}} =$

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-0.0295 rad, $\beta_{\rm trim}=p_{\rm trim}=q_{\rm trim}=r_{\rm trim}=\varphi_{\rm trim}=\delta_{a_{\rm trim}}=\delta_{r_{\rm trim}}=0$, $|\delta_e|\le 0.3$ rad, $|\delta_a|\le 0.3$ rad, $|\delta_r|\le 0.3$ rad flight conditions is taken as follows.[18]

Longitudinal dynamics:

$$\dot\theta = q - 2q\sin^2(\varphi/2) - r\sin\varphi$$
$$\dot\alpha = -\beta p + 0.0427\cos\theta\cos\varphi + 0.083589 - 1.15\alpha + 0.9937q - 0.177\delta_e$$
$$\dot q = 0.9586pr - 0.0833(r^2 - p^2) - 1.94166 + 3.724\alpha - 1.26q - 19.5\delta_e \tag{1}$$

Lateral dynamics:

$$\dot\beta = -0.9973r + \alpha p + 0.0427\cos\theta\sin\varphi - 0.2971\beta + 0.000851p + 0.03723\delta_r + 0.002466\delta_a$$
$$\dot\varphi = p + q\sin\varphi\tan\theta + r\cos\varphi\tan\theta$$
$$\dot p = -0.1345pq - 0.8225qr - 53.48\beta - 4.3242p - 0.2237r - 50.933\delta_a + 10.177\delta_r$$
$$\dot r = -0.7256pq + 0.1345qr + 17.671\beta + 0.2339p - 0.6487r + 4.125\delta_a - 6.155\delta_r \tag{2}$$

Actuators dynamics:

$$\dot\delta_e = 20(u_1-\delta_e),\qquad \dot\delta_a = 20(u_2-\delta_a),\qquad \dot\delta_r = 20(u_3-\delta_r) \tag{3}$$

The system outputs under control will be the flight-path angle $\gamma=\theta-\alpha$, the sideslip angle β, and the roll angle φ. The system (1–3) will be forced by unknown but bounded disturbances: the unmatched disturbance will represent a casual wind gust and will be subjoined to δ_e, δ_a, δ_r in an additive manner to simulate the correspondent disturbing aerodynamic forces and torques. The matched disturbance will represent the actuator imperfections and will be subjoined to u_1, u_2, u_3 inputs.

The control goal is to track the profiles given in real time for the flight-path angle and the roll angle, holding the sideslip angle at zero in the same time. As known, the flight-path angle is a nonminimum phase output of the conventional-scheme aircraft with respect to the elevator deflection as an input. In the next section we show that the main problem in output tracking for this particular system exists in the longitudinal channel because the rest part of input-output dynamics is of minimum phase.

III. System Basis Transformation and Nonminimum Phase Condition

First, we intend to transform the linear part of the systems (1) and (2) to the normal canonical form.[1] It will be more convenient to analyze the system and to distinguish the zero dynamics,[1] which are supposed to be unstable.

Considering the flight-path angle γ as an output and δ_e as a control input for the longitudinal dynamics, we select the nonsingular linear transformation as follows:

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-6.464 & 7.191 & -0.065\\ 7.173 & -6.484 & 0.059\\ 1 & -1 & 0\end{bmatrix}\cdot\begin{bmatrix}\theta\\ \alpha\\ q\end{bmatrix} \tag{4}$$

The system (1) is transformed to the form

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\end{bmatrix}=\begin{bmatrix}-11.707 & 0 & -75.666\\ 0 & 11.141 & -79.908\\ 0.723 & 0.907 & -1.844\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\0.177\end{bmatrix}\delta_e + \begin{bmatrix}-6.464 & 7.191 & -0.065\\ 7.173 & -6.484 & 0.059\\ 1 & -1 & 0\end{bmatrix}\cdot\begin{bmatrix}-2q\sin^2(\varphi/2) - r\sin\varphi\\ -\beta p + 0.0427\cos\theta\cos\varphi + 0.08\\ 0.9586pr - 0.0833(r^2-p^2) - 1.94\end{bmatrix} \tag{5}$$

where the linear part of the system (5) is in the normal form in the sence that $\dot\gamma=\dot x_3 \propto \delta_e$ represents the input-output dynamics and, therefore, (x_1, x_2) are the states of the internal dynamics.[1]

Then, the zero dynamics in longitudinal channel under the condition $\gamma=x_3=0$, $\varphi=\beta=0$ are obtained as

$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix}=\begin{bmatrix}-11.707 & 0\\ 0 & 11.141\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}-6.464 & 7.191\\ 7.173 & -6.484\end{bmatrix}\cdot\begin{bmatrix}0\\ 0.04\cos\theta + 0.08\end{bmatrix} \tag{6}$$

The second term in the right part of this expression is bounded and has no influence on stability, whereas the linear part is obviously unstable. Hence, we have $\gamma=x_3$ as a nonminimum phase output with respect to δ_e as a control input. The system (5) has relative degree $r=1$ and second-order internal dynamics,[1] which are presented by the first two equations, where x_3 should be replaced by a reference output profile. The linear part of the internal dynamics are unstable with one eigenvalue at $\lambda=11.141$. So, designing the control to provide zero-output $\gamma=x_3=0$ and ignoring zero dynamics (6) will require infinite control input to compensate for unstable states of the system (5). This fact causes problems in the control design for any methods applied to nonminimum phase systems including sliding mode control, as it is shown in Sec. IV.

For the lateral dynamics we consider the sideslip angle β and the roll angle φ as the outputs, δ_a, δ_r as the control inputs, and selecting the nonsingular transformation as follows:

$$\begin{bmatrix}x_4\\x_5\\x_6\\x_7\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 271.5125 & 0 & 0.1688 & 1.9213\end{bmatrix}\begin{bmatrix}\beta\\ \varphi\\ p\\ r\end{bmatrix} \tag{7}$$

From Eqs. (2), (4), and (7) we obtain the zero dynamics in lateral channel under the conditions $\gamma=0$, $\varphi=x_4=0$, $\beta=x_5=0$, and $\tan\theta\ll 1$, then $\dot x_5\approx x_6$, $x_6=0$ as

$$\dot x_7 = [-141.6 + 0.52\cdot(-7.86x_1 + 8.29x_2)]x_7 \tag{8}$$

Thus, if the states x_1, x_2 of the longitudinal channel are bounded within domain $(-7.86x_1 + 8.29x_2) < 272.31$, then the zero dynamics of the lateral channel (8) are stable. As far as the zero dynamics (6) are uncoupled from Eq. (8) and the system (1) is separate from Eq. (2) control input, the problem of nonminimum phase output is localized within the longitudinal channel only. Given the stable closed-loop longitudinal dynamics, the lateral dynamics (2) are of minimum phase for selected outputs. Consequently, to design the controller to track γ, φ, β, the problem of nonminimum phase output must be resolved in the longitudinal channel only.

In the next section we demonstrate the DSM technique for a class of MIMO nonlinear nonminimum phase systems. In Sec. V this technique is employed to design an SMC for the system motion in a longitudinal (pitch) plane.

IV. Dynamic Sliding Manifold Design: General Case

The system (5) has the state-space dimension $n=3$, input/output dimension $m=1$, and relative degree[1] $r=1$. We can observe that $n-m>m$ and $r=m$ in this case. To design a generic algorithm for a class of nonminimum phase systems, we generalize the system (5) to the form where n is arbitrary and $1\le m<n/2$, $r=m$, r is the total relative degree.[1] The generalized system in the normal form is

$$\dot\psi = A_{11}\psi + A_{12}\xi + f_1(t)$$
$$\dot\xi = A_{21}\psi + A_{22}\xi + B_2u + D_2F_1(\psi,\xi,t),\qquad y=\xi \tag{9}$$

where ψ is an $(n-m)$ vector of the linear internal dynamics, y, ξ, $u\in\Re^m$, $|B_2|\ne 0$, $F_1(\cdot)$ is a bounded nonlinear uncertain term, $f_1(t)$ is a bounded smooth external disturbance with zero high derivatives beginning with kth, $f_1^{(i)}(t)\equiv 0$, $i\ge k$ almost everywhere. The system (9) is assumed to be of nonminimum phase, meaning that matrix A_{11} is non-Hurwitz. Additionally, we assume that the pair (A_{11}, A_{12}) is completely controllable; matrix A_{12} is of full rank.

The problem is to asymptotically track the profile $y^*(t)$ given in real time, with the finite known number k of nonzero time derivatives $y^{*(i)}(t) \equiv 0$, $i \geq k$ (almost everywhere), in the presence of the unmatched disturbance $f_1(t)$ of the same property.

Nonminimum phase tracking via conventional SMC design cannot be achieved. Indeed, consider a conventional sliding surface for the system (9)

$$\sigma = e, \qquad e = y^* - y$$

and conventional SMC law[2] as

$$u = \hat{u}_{\text{eq}} + B_2^{-1} R \cdot \text{SGN}(\sigma), \qquad \hat{u}_{\text{eq}} = B_2^{-1}(-A_{21}\psi - A_{22}\xi) \tag{10}$$

where $R = \text{diag}\{\rho_i\}$, $\text{SGN}(\sigma) = [\text{sgn}(\sigma_1), \text{sgn}(\sigma_2), \ldots, \text{sgn}(\sigma_m)]^T$, $i = \overline{1, m}$,

$$\rho_i > \max_{t,(\psi,\xi)\in\Omega\subset\Re^n} \left|\sum_{j=1}^{m} b_{ij}\Delta F_j\right| + \tilde{\rho}_i$$

$\tilde{\rho}_i > 0$, $i = \overline{1, m}$, $\Delta F = \dot{y}^*(t) - D_2 F_1(\psi, \xi, t)$ and b_{ij} are elements of matrix B_2. Applying Lyapunov stability analysis to this case and selecting a Lyapunov function candidate as $V = \frac{1}{2}\sigma^T\sigma$, we can derive that $\forall(\psi, \xi)_{t=0} \in \Omega \subset \Re^n$

$$\dot{V} \leq -\left(\sum_{i=1}^{m} \tilde{\rho}_i|\sigma_i|\right) < 0$$

under the control law (10). Hence, the sliding mode $\sigma = 0$ will exist, and $e = 0$ in the sliding mode. The system (9) motion in the sliding mode is described by

$$e = 0, \qquad \dot{\psi} = A_{11}\psi + A_{12}y^*(t) + f_1(t)$$

The ψ dynamics (internal dynamics of the closed-loop system) are unstable, and therefore the sliding mode can be achieved only under unbounded control law for $\lim_{t\to\infty} \|\hat{u}_{\text{eq}}(\psi, t)\| = \infty$.

SMC control must be designed 1) to provide the output tracking-error dynamics with given eigenvalue placement and 2) to stabilize the closed-loop internal dynamics. In this case, the sliding mode will exist under bounded control law.

To provide the stable internal dynamics of the closed-loop system, DSM design is accomplished as follows.

First, we select the sliding surface

$$\sigma = e + C\psi + v = 0 \tag{11}$$

where $e = y^* - y$, $\sigma, e, v \in \Re^m$, $C \in \Re^{m\times(n-m)}$, C, v are to be determined.

The system (9) motion in the sliding manifold (11) is derived as

$$\dot{\psi} = (A_{11} + A_{12}C)\psi + A_{12}v + \left[A_{12}y^*(t) + f_1(t)\right] \tag{12}$$

$$-e = C\psi + v \tag{13}$$

Second, using linear subspace decomposition technique,[19] we build a nonsingular linear transformation $M \in \Re^{(n-m)\times(n-m)}$ to transform the system (12) to the form given in the following lemma.

Lemma: Given the nonsingular linear transformation

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = M\psi, \qquad z_1 \in \Re^{n-2m}, \quad z_2 \in \Re^m \tag{14}$$

where

$$M = \begin{bmatrix} M_1^\# \\ M_2 \end{bmatrix}, \qquad M^{-1} = \begin{bmatrix} M_1 & M_2^\# \end{bmatrix}, \qquad M_2^\# = M_2^T\left(M_2M_2^T\right)^{-1}$$

$$M_1^\# = \left(M_1^TM_1\right)^{-1}M_1^T$$

1) Matrix M_1 is selected such that columns of $M_1 \in \Re^{(n-2m)\times m}$ are $n - 2m$ column eigenvectors of matrix A_{11}; matrix M_1 is built such that

$$M_1 = \begin{bmatrix} M_1' \\ M_1'' \end{bmatrix}, \qquad |M_1''| \neq 0$$

2) Matrix $M_2 \in \Re^{m\times(n-m)}$ is calculated as $M_2 = [I_{m\times m} \vdots -M_1'M_1''^{-1}]$ so that $M_2M_1 \equiv 0$, i.e., the column-range space of matrix M_1 is in the null space of matrix M_2.

3) Matrix C in Eq. (11) is selected such that $C = M_2$.

Then the systems (11) and (12) are presented in the new basis as

$$\begin{gathered} \dot{z}_1 = \tilde{A}_{11}z_1 + \tilde{A}_{12}z_2 + \tilde{B}_1v + \phi_1(t) \\ \dot{z}_2 = \tilde{A}_{22}z_2 + \tilde{B}_2e + \phi_2(t), \qquad e = -(z_2 + v) \\ \tilde{A}_{11} = M_1^\#A_{11}M_1, \qquad \tilde{A}_{12} = M_1^\#A_{11}M_2^\# + A_{12} \\ \tilde{A}_{22} = M_2A_{11}M_2^\# \\ MA_{12} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \qquad M(A_{12}y^* + f_1) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} \end{gathered} \tag{15}$$

Proof: See Appendix A.

We obtain the system (15) in the uncoupled format. If z_1 dynamics are stable (matrix $\tilde{A}_{11}$ is Hurwitz) given bounded input $[\tilde{A}_{12}z_2 + \tilde{B}_1v + \phi_1(t)]$, then considering $v = v(z_2, e)$ as a virtual control the problem now is to stabilize the second equation in Eq. (15) as well as to provide output tracking error e convergence to zero.

The following theorem gives the solution to the last problem.

Theorem 1a: If $\exists M \in \Re^{(n-m)\times(n-m)}$ such that the Eqs. (12) and (13) of the system (9) motion in the sliding manifold (11) are transformed by Eq. (14) to the form (15), where matrix $\tilde{A}_{11}$ is Hurwitz, and $|\tilde{A}_{22}| \neq 0$, then $\exists P_k, P_{k-1}, \ldots, P_0 \in \Re^{m\times m}$, such that the following is true:

1) Any real-time reference profile $y^*(t)$, $y^{*(i)}(t) \equiv 0$, $i \geq k$, will be asymptotically followed in sliding mode in the DSM (11), where

$$v = (P_k - I_{m\times m})e + \int\left[P_{k-1}e + \int\left(P_{k-2}e + \cdots + \int P_0e\,\mathrm{d}\tau\right)\cdots\mathrm{d}\tau\right]\mathrm{d}\tau \tag{16}$$

in accordance with the following system of differential equations for output tracking-error:

$$e^{(k+1)} + c_ke^{(k)} + \cdots + c_1\dot{e} + c_0e = 0 \tag{17}$$

where the set of numbers $(c_k, c_{k-1}, \ldots, c_0)$ is to be specified to provide given eigenvalue placement.

2) Given the output tracking-error dynamics (17), the set of matrices $P_k, P_{k-1}, \ldots, P_0 \in \Re^{m\times m}$ is calculated as

$$\begin{gathered} P_0 = -c_0(\tilde{A}_{22})^{-1}P_k \\ P_1 = -\left[c_0(\tilde{A}_{22})^{-2} + c_1(\tilde{A}_{22})^{-1}\right]P_k \\ \vdots \\ P_{k-1} = -\left[c_0(\tilde{A}_{22})^{-k} + \cdots + c_{k-1}(\tilde{A}_{22})^{-1}\right]P_k \\ P_k = \left[c_0(\tilde{A}_{22})^{-k} + \cdots + c_{k-1}(\tilde{A}_{22})^{-1} + \tilde{A}_{22} + c_kI\right]^{-1}\tilde{B}_2 \end{gathered} \tag{18}$$

Proof: See Appendix B.

Theorem 1b: The sliding mode in DSMs (11) and (16) exists under the bounded control law

$$u = \hat{u}_{\text{eq}} + B_2^{-1}R \cdot \text{SGN}(\sigma) \tag{19}$$

where $\hat{u}_{\rm eq} = B_2^{-1}(-A_{21}\psi - A_{22}\xi)$, $R = {\rm diag}\{\rho_i\}$, ${\rm SGN}(\sigma) = [{\rm sgn}(\sigma_1), {\rm sgn}(\sigma_2), \ldots, {\rm sgn}(\sigma_m)]^T$, $i = \overline{1, m}$,

$$\rho_i > \max\left|\sum_{j=1}^{m} b_{ij}\Delta F_j\right|$$

$i = \overline{1, m}$, $\Delta F = \dot{y}^* + C\dot{\psi} + \dot{v} - D_2F_1(\cdot)$ and b_{ij} are elements of matrix B_2.

Proof: See Appendix C.

Thus, the system (9) motion in the DSMs (11) and (16) accomodates the unmatched disturbance $f_1(t)$, piecewise presented by polynomial splines. Tracking is invariant to matched disturbances and provides asymptotic tracking-error convergence to zero with given eigenvalue placement.

V. DSM Design for Flight-Path Angle Tracking

The zero dynamics (6) in the longitudinal channel matches the zero dynamics of the model (9) because a nonlinear state-dependent term in Eq. (6) is globally bounded and has no influence on stability. So, if we succeed to stabilize the linear part of the zero dynamics (6), then tracking in the longitudinal channel will be provided with locally stable internal dynamics. Thus, we adopt the model (9) to design a tracking controller for the system (5). The internal dynamics in the longitudinal channel are modeled as

$$\dot{\psi} = A_{11}\psi + A_{12}(y^* - e) + f_1(t) \tag{20}$$

which have the form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-11.707 & 0\\ 0 & 11.141\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-75.666\\ -79.908\end{bmatrix}(y^* - e) + f_1(t) \tag{21}$$

where $y^*(t)$ is the output reference profile for the flight-path angle. The system (21) is in uncoupled format, and x_1 dynamics is bounded given the bounded input. So, the transformation (14) is identity

$$M = I_{2\times 2}, \qquad C = M_2 = [0 \quad 1]$$

The DSM (11) has the form

$$\sigma = e + x_2 + v = 0 \tag{22}$$

According to the lemma, the second equation to be stabilized in the system (15) has the form ($m = 1$)

$$\dot{x}_2 = 11.141x_2 + 79.908e + \varphi_1(t) \tag{23}$$

where $\varphi_1(t) = -79.908y^*(t) + f_1(t)$. We accept the following model for uncertain disturbance $\varphi_1(t)$:

$$\dddot{\varphi} = 0$$

almost everywhere. So, $k = 3$, and according to theorem 1 we obtain the fourth-order tracking-error dynamics of the form (17), which is selected to be

$$e^{(4)} + 4.2\dddot{e} + 13.6\ddot{e} + 21.6\dot{e} + 16e = 0 \tag{24}$$

where the eigenvalues have been chosen according to integral time average error criterion with the time constant $\tau \approx 3$ s to produce $\omega_n = 2$ rad/s. In this case $A_{22} = a_{22} = 11.141$, $\tilde{B}_2 = \tilde{b}_2 = 79.908$, $c_3 = 4.2$, $c_2 = 13.6$, $c_1 = 21.6$, $c_0 = 16$, and from Eq. (18) we have

$$p_3 = \left(c_0a_{22}^{-3} + c_1a_{22}^{-2} + c_2a_{22}^{-1} + a_{22} + c_3\right)^{-1}\tilde{b}_2 = 4.8$$

$$p_2 = -\left(c_0a_{22}^{-3} + c_1a_{22}^{-2} + c_2a_{22}^{-1}\right)p_3 = -6.71$$

$$p_1 = -\left(c_0a_{22}^{-2} + c_1a_{22}^{-1}\right)p_3 = -9.87$$

$$p_0 = -c_0a_{22}^{-1}p_3 = -6.85$$

Finally, using Eqs. (16) and (22), we obtain the following DSM:

$$\sigma = x_2 + 4.8e - \int\left[6.71e + \int\left(9.87e + 6.85\int e\,{\rm d}t\right){\rm d}t\right]{\rm d}t$$

$$x_2 = 7.173\theta - 6.484\alpha + 0.059q, \qquad e = y^*(t) - \gamma \tag{25}$$

Using Eq. (19), we build a control law as

$$\delta_e = -(1.844/0.177)(\vartheta - \alpha) + (\rho/0.177)\,{\rm sgn}(\sigma) \tag{26}$$

where ρ is to be selected to compensate for cumulative uncertainty of equivalent control for the sliding mode $\rho = 0$. The following condition should be met:

$$\rho > \big|\dot{y}^* + 11.141x_2 + 79.908e + \varphi_1(t) - 0.723x_1 - 0.907x_2 - D_2F_1(\cdot, t)\big|$$

For simulations we accept $\rho = 0.25$.

Finally, in order to avoid the actuator chattering, the SMC (26) is designed in the continuous finite-reaching-time format[17] as

$$\delta_e = -10.418(\vartheta - \alpha) + 1.41\cdot|\sigma|^{0.3}\,{\rm sgn}(\sigma) \tag{27}$$

Because the actuator dynamics (3) are much faster then the compensated tracking-error dynamics, the control law (27) is applied to the input of actuators (3), that is,

$$u_1 = -10.418(\vartheta - \alpha) + 1.41\cdot|\sigma|^{0.3}\,{\rm sgn}(\sigma) \tag{28}$$

As we will see further, the control law (28) produces smooth elevator deflection within the prescribed limit $|\delta_e| \leq 0.3$ rad. Robustness analysis can be made of the domain of stability for the original nonlinear system (5), accepting perturbations to critical parameters a_{22}, $\tilde{b}_2$ and considering the unmatched disturbance to be a state-dependent function $\varphi_1 = \varphi_1(x, t)$ (see, for example, pp. 22 and 23 in Ref. 20).

VI. DSM Controller for Lateral Motion

To complete the SMC design for the system (1–3), we will derive an SMC for output tracking in the minimum phase lateral channel, assuming that only output information is available. Here, we will employ the DSM technique to avoid numerical differentiation or derivative estimation procedure, which otherwise is necessary[21] to design a sliding manifold for the systems with relative degree $r \geq 2$.

The nonlinear terms in the system (2) are bounded within any given bounded domain around origin in the state space, given stable closed-loop dynamics in longitudinal motion. For the purpose of control design, we will present them by a vector of uncertain bounded disturbances $w(t)$, $w \in \Re^4$. Then, we write the system (2) as

$$\begin{bmatrix}\dot{\beta}\\ \dot{\varphi}\\ \dot{p}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}-0.2971 & 0.0427 & 8.51\cdot 10^{-4} & -0.9973\\ 0 & 0 & 1 & 0\\ -53.48 & 0 & -4.3242 & -0.2237\\ 17.671 & 0 & 0.2339 & -0.6487\end{bmatrix}\cdot\begin{bmatrix}\beta\\ \varphi\\ p\\ r\end{bmatrix} + \begin{bmatrix}0.002466 & 0.03723\\ 0 & 0\\ -50.933 & 10.177\\ 4.125 & -6.155\end{bmatrix}\cdot\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix} + w(t) \tag{29}$$

where we have the sideslip angle β and the roll angle φ as the outputs, δ_a, δ_r as the control inputs, and $w(t)$ as a disturbance vector input. After the transformation (7) we can write the system (29) in the normal form

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.043 & 140.636 & 0.088 & -0.519 \\ 0 & -21.868 & -4.305 & -0.116 \\ 11.594 & 3.839 \cdot 10^{-4} & 23.846 & -141.602 \end{bmatrix} \cdot \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2.466 \cdot 10^{-3} & 0.037 \\ -50.933 & 10.177 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} + \tilde{w}(t)$$

$$y_1 = x_4, \qquad y_2 = x_5, \qquad \tilde{w}(t) = (\tilde{w}_4, \tilde{w}_5, \tilde{w}_6, \tilde{w}_7)^T \tag{30}$$

The system (30) has a total relative degree $r = 3$ and the first-order stable zero dynamics, the zero dynamics (8) of the original nonlinear system (2), are also stable, as far as the states of closed-loop longitudinal channel are bounded. One can exclude the state variable x_7 from the control design, considering it to be another external bounded disturbance.

One can notice that control input δ_a dominates in the roll angle regulation, whereas δ_r does in the sideslip angle regulation; the relative degree for the input/output pair (δ_a, y_1) is equal to 2^i; the input/output pair (δ_r, y_2) is equal to 1. We will use this fact to enforce uncoupled output error behavior for $e_1 = y_1^* - y_1$, and $e_2 = y_2^* - y_2$ via SMC with DSM, where y_1^*, y_2^* are the output reference profiles for the roll angle and the sideslip angle, respectively.

For a SISO system with relative degree r in the output tracking-error state space, the sliding mode can be achieved in the following sliding manifold[19]:

Time derivative of ith order:

$$\sigma = \sum_{i=1}^{r-1} c_i e^{(i)} = 0, \qquad c_i \in \Re, \quad e^{(i)}$$

where the derivatives $e^{(i)}$ should be observed (estimated). To eliminate steady-state constant error for SMC implementation in continuous form, one can add an integral term in the sliding surface

$$\sigma = \sum_{i=1}^{r-1} c_i e^{(i)} + c_0 \int_0^t e \, \mathrm{d}t = 0$$

We employ a DSM design for the minimum-phase system (30) to asymptotically track any real-time smooth reference profile using only output information $e_1 = y_1^* - y_1$ and $e_2 = y_2^* - y_2$. The results are presented in the following theorem.

Theorem 2: The sliding mode exists in DSM

$$\sigma_1(s) = a \cdot \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s(s + a)} e_1(s), \qquad \sigma_2(s) = \frac{\tau \cdot s + 1}{s} e_2(s) \tag{31}$$

for the system (30) under control law

$$\delta_a = -\left(\omega_n^2 / 50.933\right) e_1 - \rho_1 \operatorname{sgn}(\sigma_1)$$
$$\delta_r = (1/\tau \cdot 10.177) e_2 + \rho_2 \operatorname{sgn}(\sigma_2) \tag{32}$$

Any real-time smooth reference profiles $y_1^*(t)$, $y_2^*(t)$ will be asymptotically followed in sliding mode with specified eigenvalue placement for output tracking-error dynamics

$$\ddot{e}_1 + 2\zeta\omega_n \dot{e}_1 + \omega_n^2 e_1 = 0, \qquad \dot{e}_2 + (1/\tau) e_2 = 0 \tag{33}$$

given the set of parameters ζ, ω_n, τ to be selected.

Proof: See Appendix D.

For the simulation we accept $a = 200$, $\omega_n = 4$ rad/s, $\xi = 0.7$, $\tau = 3$ s, and $\rho_1 = \rho_2 = 0.5$. This will enforce the following output tracking-error dynamics in the sliding mode:

$$\ddot{e}_1 + 5.6\dot{e}_1 + 16 e_1 = 0, \qquad \dot{e}_2 + \tfrac{1}{3} e_2 = 0$$

Finally, similar to the procedure applied for the longitudinal channel, we have our controls for the output tracking in the lateral channel as

$$u_2 = -0.5|\sigma_1|^{0.3} \cdot \operatorname{sgn}(\sigma_1), \qquad u_3 = 0.5|\sigma_2|^{0.3} \cdot \operatorname{sgn}(\sigma_2) \tag{34}$$

where σ_1, σ_2 are the outputs of the system (30) and u_2, u_3 are the commands to the ailerons and rudder actuators, respectively.

The results of simulation of causal nonminimum-phase output tracking, using the initial nonlinear model and the control law (34), are presented in Figs. 1–9.

The aircraft is performing a maneuver in pitch plane with simultaneous lateral motion. Nonminimum-phase flight-path angle tracking process is presented in Fig. 2. Flight-path angle tracking error behavior at higher resolution is presented in Fig. 3 together with unmatched disturbance function, which has been subjoined to

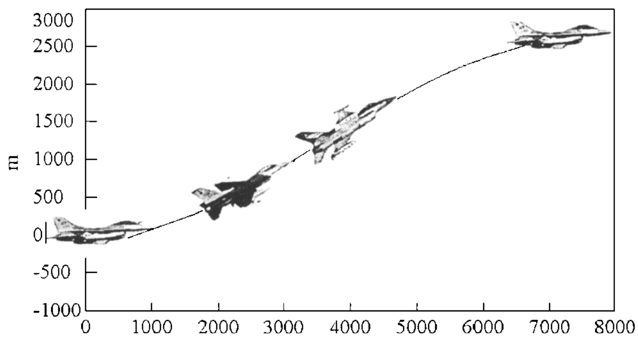

Fig. 1 Trajectory in pitch plane (the zero altitude corresponds to 3000 m).

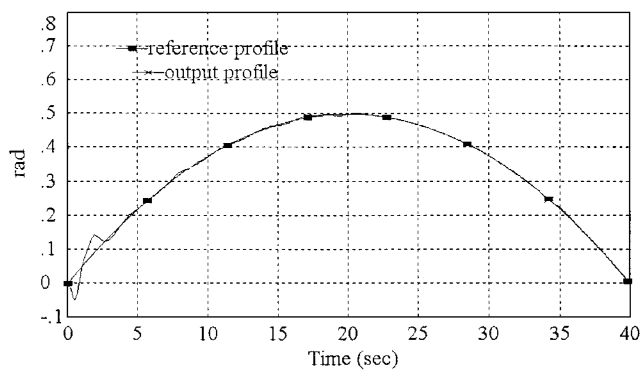

Fig. 2 Flight-path angle tracking.

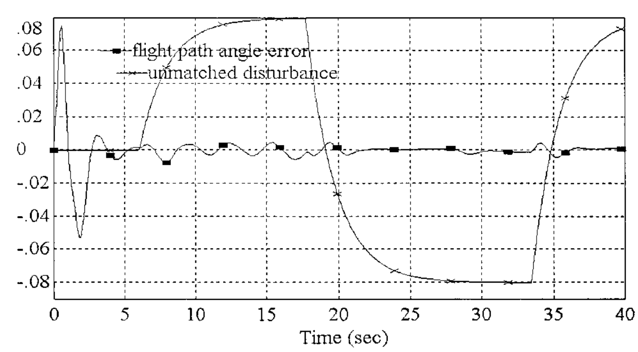

Fig. 3 Flight-path angle tracking error and the unmatched disturbance term.

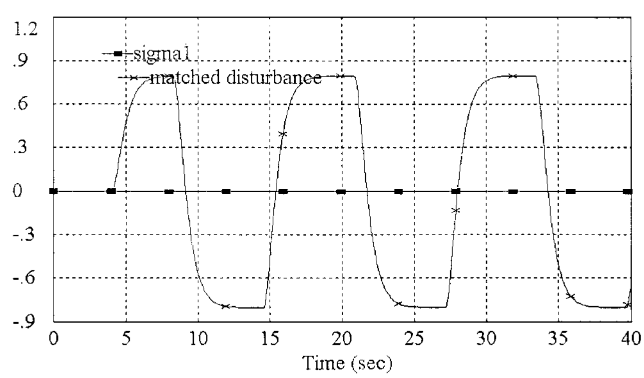

Fig. 4 Sliding surface σ and the matched disturbance term.

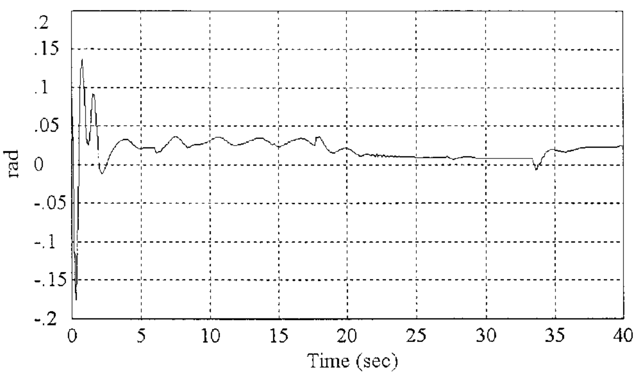

Fig. 5 Elevator deflection δ_e.

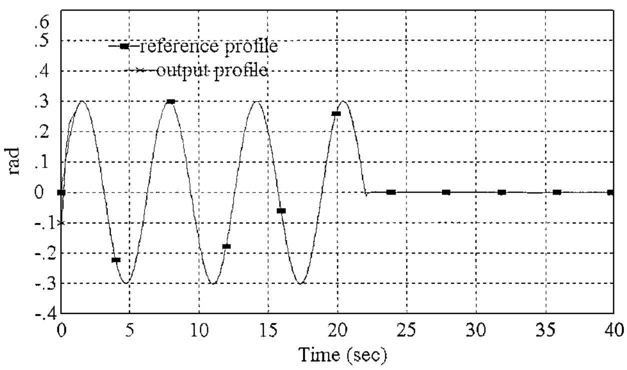

Fig. 6 Roll angle tracking.

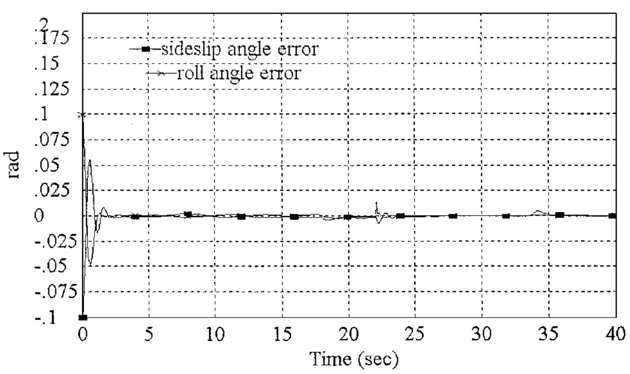

Fig. 7 Roll angle tracking error and sideslip angle stabilization error.

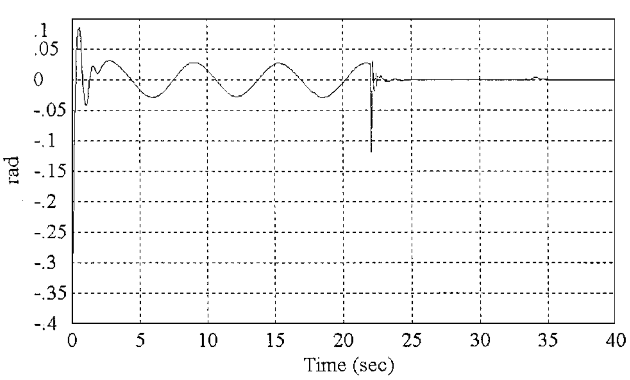

Fig. 8 Ailerons deflection.

δ_e in an additive manner. We can observe that the transient from initial conditions is over to the fifth second. We see a little ripple of error between 7–22 s as a result of the sinusoidal tracking process in lateral channel, which cross influences pitch motion in the nonlinear model (1, 2) because the controller can eliminate the influence of two time derivatives of uncertain terms only. We can see that the controller completely compensates for a constant component in unmatched disturbance function (22–33 s), and between 33–37 s

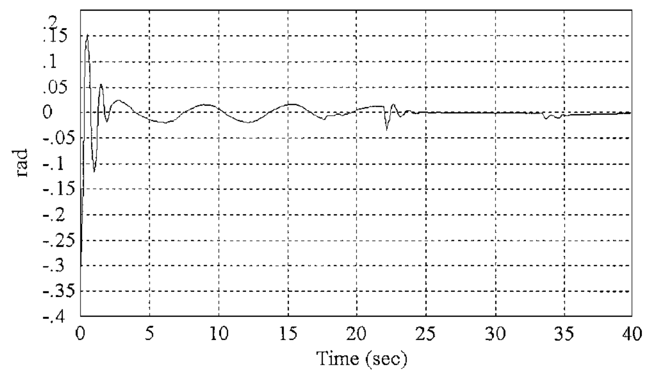

Fig. 9 Rudder deflection.

the error converges quickly to zero under the parabolic shape of disturbance function behavior and after the point of discontinuity in disturbance function derivative. Matched disturbance, which has been subjoined to u_1, u_2, u_3 inputs, modeling actuator imperfections does not affect sliding mode $\sigma = 0$ at all, as it can be seen from Fig. 4. Practically perfect tracking in lateral channel is illustrated in Figs. 6 and 7. Control inputs within prescribed limits are observed in Figs. 5, 8, and 9.

VII. Conclusions

The causal nonlinear nonminimum-phase output tracking problem for F-16 aircraft is addressed via sliding mode control. A sliding mode controller has been designed to provide robust tracking the system with matched nonlinear terms as well as matched and unmatched disturbances using dynamic sliding manifold technique. DSM has been employed to stabilize internal dynamics of a nonminimum-phase system and to design a controller without separate estimation of output tracking-error derivatives. Such a controller is shown to be insensitive to matched disturbances and nonlinearities and can accomodate unmatched disturbances as well. The proposed control scheme allows the cancellation of the error from a causal reference input with zero high derivatives.

Appendix A: Proof of Lemma

We define

$$M = \begin{bmatrix} M_1^{\#} \\ M_2 \end{bmatrix}$$

matrix M_1 is selected such that columns of $M_1 \in \Re^{(n-2m)\times m}$ are $n-2m$ column eigenvectors of matrix A_{11}. Matrix M_1 is of full rank, and we build it such that

$$M_1 = \begin{bmatrix} M_1' \\ M_1'' \end{bmatrix}, \qquad |M_1''| \neq 0$$

then Moore–Penrose generalized inverse of M_1 is $M_1^{\#} = (M_1^T M_1)^{-1} M_1^T$. Matrix $M_2 \in \Re^{m\times(n-m)}$ is selected so that $M_2 M_1 \equiv 0$, i.e., the column-range space of matrix M_1 is in the null space of matrix M_2. Matrix M_2 can be calculated as $M_2 = [I_{m\times m} \vdots -M_1' M_1''^{-1}]$. Thus, matrix M defines a nonsingular linear transformation. The inverse transformation is calculated as $M^{-1} = [M_1 \;\; M_2^{\#}]$, $M_2^{\#} = M_2^T (M_2 M_2^T)^{-1}$ because we have the following identities $M_2 M_2^{\#} = I_{m\times m}$, $M_1^{\#} M_1 = I_{(n-2m)\times(n-2m)}$, $M_2 M_1 \equiv 0$, $M_1^{\#} M_2^{\#} \equiv 0$; therefore, $M M^{-1} = I_{(n-m)\times(n-m)}$.

Because matrix C in Eq. (11) is selected such that $C = M_2$, we have

$$M(A_{11} + A_{12}C)M^{-1} = \begin{bmatrix} M_1^{\#} A_{11} M_1 & M_1^{\#} A_{11} M_2^{\#} + M_1^{\#} A_{12} \\ M_2 A_{11} M_1 & M_2 A_{11} M_2^{\#} + M_2 A_{12} \end{bmatrix}$$

$$CM^{-1} = [0 \quad I_{m\times m}]$$

As far as the column range space of matrix M_1 is invariant with respect to matrix A_{11} and $M_2 M_1 \equiv 0$, then $M_2 A_{11} M_1 \equiv 0$

as well. Using identity $(M_2 A_{11} M_2^{\#} + M_2 A_{12}) z_2 + M_2 A_{12} v = M_2 A_{11} M_2^{\#} z_2 + M_2 A_{12} e$, the system (11) and (12) is presented in the new basis as

$$\dot{z}_1 = \tilde{A}_{11} z_1 + \tilde{A}_{12} z_2 + \tilde{B}_1 v + \phi_1(t)$$

$$\dot{z}_2 = \tilde{A}_{22} z_2 + \tilde{B}_2 e + \phi_2(t), \qquad e = -(z_2 + v)$$

$$\tilde{A}_{11} = M_1^{\#} A_{11} M_1, \qquad \tilde{A}_{12} = M_1^{\#} A_{11} M_2^{\#} + A_{12}$$

$$\tilde{A}_{22} = M_2 A_{11} M_2^{\#}$$

$$M A_{12} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix}, \qquad M\left(A_{12} y^* + f_1\right) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}$$

Appendix B: Proof of Theorem 1a

If matrix $\tilde{A}_{11}$ is Hurwitz, then the problem to stabilize the rest part of the system (15) is solved as follows. Combining the third equation in Eq. (15) with Eq. (16) and differentiating the result k times, we get

$$-z_2^{(k)} = P_k e^{(k)} + P_{k-1} e^{(k-1)} + \cdots + P_0 e \tag{B1}$$

The second equation in Eq. (15) can be rewritten as

$$\left(I \frac{\partial}{\partial t} - \tilde{A}_{22}\right) z_2 = \tilde{B}_2 e + \varphi_2(t) \tag{B2}$$

One can decouple the system (B1) and (B2) for e as

$$e^{(k+1)} + P_k^{-1}(\tilde{B}_2 + P_{k-1} - \tilde{A}_{22} P_k) e^{(k)} + \cdots$$
$$+ P_k^{-1}(P_0 - \tilde{A}_{22} P_1)\dot{e} + P_k^{-1}(-\tilde{A}_{22} P_0) e = -P_k^{-1} \varphi_2^{(k)}$$

As far as $\varphi_2(t) = M_2 A_{12} y^*(t) + M_2 f_1(t)$, then $\varphi_2^{(k)} = 0$ almost everywhere.

We want to have tracking-error dynamics in the uncoupled format as

$$e^{(k+1)} + c_k e^{(k)} + \cdots + c_1 \dot{e} + c_0 e = 0$$

where the set of numbers $(c_k, c_{k-1}, \ldots, c_1, c_0)$ is selected according to desired eigenvalue placement. Then, if $|\tilde{A}_{22}| \neq 0$, we calculate the set of matrices $P_k, P_{k-1}, \ldots, P_0 \in \Re^{m \times m}$ as

$$P_0 = -c_0(\tilde{A}_{22})^{-1} P_k$$

$$P_1 = -\left[c_0(\tilde{A}_{22})^{-2} + c_1(\tilde{A}_{22})^{-1}\right] P_k$$

$$\vdots$$

$$P_{k-1} = -\left[c_0(\tilde{A}_{22})^{-k} + \cdots + c_{k-1}(\tilde{A}_{22})^{-1}\right] P_k$$

$$P_k = \left[c_0(\tilde{A}_{22})^{-k} + \cdots + c_{k-1}(\tilde{A}_{22})^{-1} + \tilde{A}_{22} + c_k I\right]^{-1} \tilde{B}_2$$

Output tracking error e will asymptotically converge to zero. Solving the system (B1) and (B2) for z_2, we can prove bounded z_2 behavior with the same characteristic polynomial as well, provided $|\tilde{A}_{22}| \neq 0$. Thus, the system (9) motion in the DSMs (11) and (16) provides asymptotically stable output tracking-error dynamics and bounded compensated internal dynamics.

Appendix C: Proof of Theorem 1b

Similar to the Lyapunov analysis made in Sec. IV to derive the existence condition for the control law (10), conditions of theorem 1b for existence and boundedness of the control law (19), given bounded z_2 behavior, can be derived.

Appendix D: Proof of Theorem 2

We can present the first equation in Eq. (31) as

$$\ddot{\sigma}_1 + a\dot{\sigma}_1 = a\left[\varphi(x,t) + \omega_n^2 e_1 + 50.933\delta_a\right] \tag{D1}$$

where $\varphi(x,t) = \ddot{y}_1^* + 21.868 x_5 + 4.305 x_6 + 0.116 x_7 - 10.177\delta_r - \tilde{w}_6(t) + 2\xi\omega_n(\dot{y}_1^* - x_6)$. Substituting the control law (28) for δ_a into Eq. (D1), we obtain the following closed-loop system:

$$\ddot{\sigma}_1 + a\dot{\sigma}_1 = a[\varphi(x,t) - 50.933\rho_1 \operatorname{sgn}(\sigma_1)]$$

$\exists \infty > \tilde{L}(\rho_1) > 0$, such that $-\tilde{L}\operatorname{sgn}(\sigma_1)$ will be the function majoring $\varphi(x,t) - 50.933\rho_1 \operatorname{sgn}(\sigma_1)$ for any compact bounded domain in the system (D1) state space and given bounded $\ddot{y}_1^*, \dot{y}_1^*, y_1^*, \tilde{w}_6(t), \delta_r(\cdot)$.

Using $-\tilde{L}\operatorname{sgn}(\sigma_1)$, one can build equation for $\tilde{\sigma}_1$, which is majoring σ_1

$$\ddot{\tilde{\sigma}}_1 + a\dot{\tilde{\sigma}} + \tilde{L}\operatorname{sgn}(\tilde{\sigma}_1) = 0 \tag{D2}$$

Thus, if the majorant $\tilde{\sigma}_1 \to 0$, then $\sigma_1 \to 0$ as well. Let us introduce a Lyapunov function candidate as follows: $V = \dot{\tilde{\sigma}}_1^2/2 + \tilde{L}|\tilde{\sigma}_1|$, $\forall \tilde{\sigma}_1 \neq 0, \dot{\tilde{\sigma}}_1 \neq 0: V > 0$, then $\dot{V} = -a\dot{\tilde{\sigma}}_1^2$. Thus, if $a > 0$, then $\forall \tilde{\sigma}_1$, $\forall \dot{\tilde{\sigma}}_1 \neq 0: \dot{V} < 0$, and the system (D2) asymptotically attains the domain $\dot{\tilde{\sigma}}_1 = 0$ in the $(\tilde{\sigma}_1, \dot{\tilde{\sigma}}_1)$ state space. Analyzing the system (D2), we can conclude that the only equilibrium point in this domain is $\tilde{\sigma}_1 = \dot{\tilde{\sigma}}_1 = 0$. Hence, $\tilde{\sigma}_1$ asymptotically converges to zero, and so does σ_1 if $50.933\rho_1 > |\varphi(x,t)|$. We can apply the same approach to prove the existence of the sliding mode in the DSM $\sigma_2 = 0$ and derive the analogous conditions for ρ_2.

After the sliding manifold is reached, we have the error dynamics (33) in the sliding mode $\sigma_1 = \sigma_2 = 0$, according to Eq. (31).

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